EQUILIBRIUM STATES OF A CYLINDRICAL SHELL WITH INITIAL DEFLECTIONS IN COMPRESSION

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In the stability analysis of a long elastic cylindrical shell under longitudinal compression the initial deflections have a substantial influence on the upper critical value of loading [1].

In the work of Koiter [2] and later of Kan [3] it was shown that initial deflection which is symmetrical with respect to the axis of the shell can significantly decrease the upper value of critical loading at the expense of change in the precritical stresses and deformed state of the shell.

In this work an analysis is carried out of the effect of initial deflections of more general form which has axially symmetric and nonsymmetric components. Since in the general case points of bifurcation are absent on curves of equilibrium states, the determination of critical loading in the investigation of the stability of the shell consists in finding the limit point which is reached in the process of successive loading of the shell. The physical significance of the limiting point is that for the given load the shell has zero strength in compression. In case of degeneracy of the first limit point (the case of large initial deflections) it is proposed to determine the value of critical loading from the condition of attainment of minimum strength of the shell in the process of loading.

Let us examine a cylindrical shell of radius R with initial deflection

$$w_0 = \tfrac{1}{2} f_0 \cos 2\alpha_1 x + \tfrac{1}{2} f_{0k} \sin \alpha_1 x \sin \beta_1 y \qquad (1)$$

In this manner the axially nonsymmetric component of initial deflection is introduced into the analysis. The initial deflection is of the form which in the solution of the buckling problem of the shell [2 and 3] is examined as an intermediate form of equilibrium.

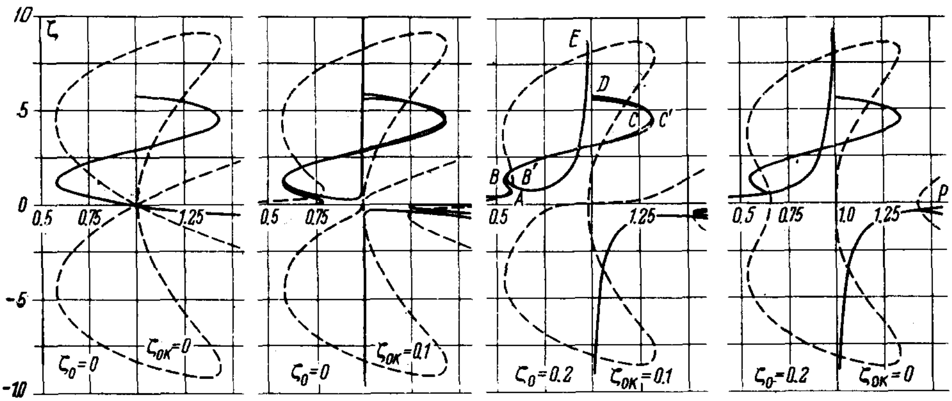


Fig. 1

The solution of nonlinear equations of a shallow shell

$$\frac{1}{B}\Delta\Delta\varphi = \frac{1}{R}(w - w_0)_{xx} + w_{xy}^2 - w_{xx}w_{yy} - (w_{0xy}^2 - w_{0xx}w_{0yy}) \qquad (2)$$

$$D\Delta\Delta(w - w_0) = -\frac{1}{R}\varphi_{xx} + \varphi_{yy}w_{xx} + \varphi_{xx}w_{yy} - 2\varphi_{xy}w_{xy}, \quad \left(B = Eh, \quad D = \frac{Eh^3}{12(1-\nu^2)}\right)$$

is sought in the form

$$w = {}^1/_2\, f \cos 2\alpha_1 x + {}^1/_2\, f_k \sin \alpha_1 x \sin \beta_1 y + f_1. \tag{3}$$

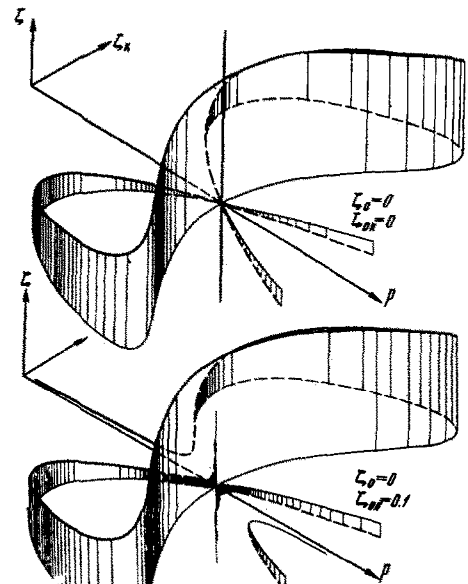
Parameters α_1 and β_1 remain the same as in the expression of the initial deflection w_0. Such an approach corresponds to an adopted calculation scheme in which the process of development of a given initial imperfection is studied.

After determination of function φ from the first equation of (2) and integrating the second equation in the sense of Galerkin, we obtain equations which determine the relative buckling deformations ζ and ζ_k of the shell in equilibrium states

$$\gamma_1(\zeta - \zeta_0) - \zeta p - \frac{1}{64\theta^2}(\zeta_k^2 - \zeta_{0k}^2) - \frac{1}{8}\gamma_4(\zeta_k - \zeta_{0k})\zeta_k + \gamma_2(\zeta\zeta_k - \zeta_0\zeta_{0k})\zeta_k = 0 \tag{4}$$

$$\gamma_3(\zeta_k - \zeta_{0k}) - p\zeta_k - \gamma_4(\zeta\zeta_k - \zeta_0\zeta_{0k}) - \gamma_4(\zeta_k - \zeta_{0k})\zeta -$$

$$- \frac{1}{4\theta^2}(\zeta - \zeta_0)\zeta_k + 8\gamma_2(\zeta\zeta_k - \zeta_0\zeta_{0k})\zeta + \frac{\alpha^2}{128}\gamma_5(\zeta_k^2 - \zeta_{0k}^2)\zeta_k = 0$$



Here

$$\gamma_1 = \frac{1}{2}\left(\alpha^2 + \frac{1}{\alpha^2}\right)$$

$$\gamma_2 = \frac{\alpha^2}{16}\left[\frac{1}{(1+9\theta^2)^2} + \frac{1}{(1+\theta^2)^2}\right]$$

$$\gamma_3 = \frac{1}{2}\left(\frac{\alpha^4}{4\theta^2\gamma_4} + \frac{4\theta^2\gamma_4}{\alpha^2}\right)$$

$$\gamma_4 = \frac{\theta^2}{(1+\theta^2)^2}, \quad \gamma_5 = \frac{1+\theta^4}{\theta^4} \tag{5}$$

$$p = \frac{\sigma}{\sigma_e}, \quad \alpha^2 = \frac{2Rh\alpha_1^2}{\lambda}$$

$$\lambda = \sqrt{3(1-\nu^2)}, \quad \theta = \frac{\alpha_1}{\beta_1}$$

$$\zeta_0 = \frac{\lambda f_0}{h}, \quad \zeta_{0k} = \frac{\lambda f_{0k}}{h}$$

$$\zeta = \frac{\lambda f}{h}, \quad \zeta_k = \frac{\lambda f_k}{h}, \quad \sigma_e = \frac{Eh}{\lambda R}$$

(h is the thickness of the shell, σ is the compression stress).

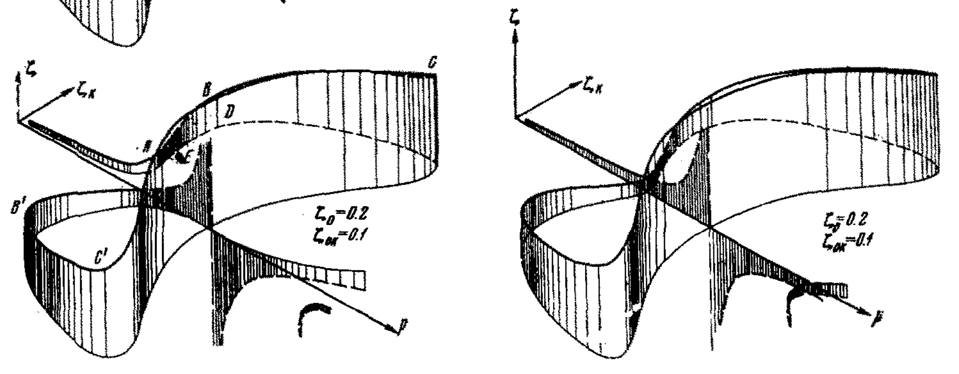


Fig. 2

Analysis of system (4) shows that the quantity ζ is uniquely determined from the first equation for given values of ζ_k and p. After elimination of ζ, the second equation is of the seventh order with respect to ζ_k. Therefore there must exist no more than seven pairs of real values ζ and ζ_k, which determine the equilibrium state of the shell. These considerations were utilized in putting together the program for solution of system (4).

In the calculation of equilibrium states it was assumed that $\alpha = \theta = 1$. Results of calculations are presented in the form of graphs in Fig. 1. Solid curves correspond to deflection ζ, dashed to ζ_k. In Fig. 2, curves of equilibrium states are presented in the space ζ, ζ_k, p.

From the obtained results it follows that, in the case when axially nonsymmetric buckling $\zeta_{0k} \neq 0$ exists, for some value p the first limit point A is reached on the line $\zeta = \zeta(p)$ and correspondingly on the line $\zeta_k = \zeta_k(p)$. The next branch AB will be unstable. With increase in p after point B the shell again falls on a stable branch BC. The line $BCDC'B'$ forms a loop. The point E is the limit point for $\zeta \to \infty$ and $\zeta_k \to 0$. We note that in all cases values of $\zeta_k \to 0$ correspond to the limit point of line ζ going to infinity. This means that in the development of large axially symmetric deformations the shell corrects initial axially nonsymmetric indentations.

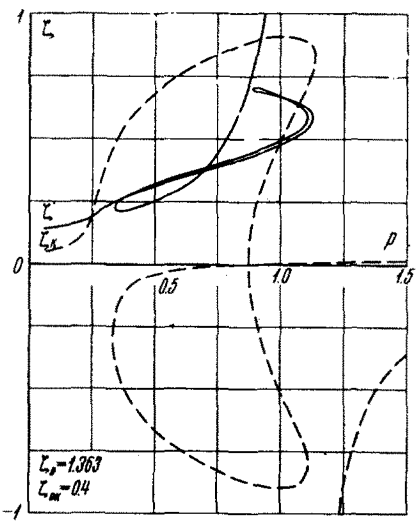


Fig. 3

On reaching the first limit point A, transition (snap-through) is possible to a stable new branch BC. This value p will be the limit of the interval of stability of the shell for given initial imperfections.

In the case when the initial axially nonsymmetric deflection is equal to zero $\zeta_{0k} = 0$ for values of p which correspond to the solutions of Koiter [2], a bifurcation point occurs (intermediate form of equilibrium, for which $\zeta_k \neq 0$).

For determination of the value of p' for which the limit point occurs the following conditions must be satisfied in addition to equilibrium equations (4)

$$\frac{dp}{d\zeta} = \frac{dp}{d\zeta_k} = 0 \tag{6}$$

The quantities $d\zeta / dp$ and $d\zeta_k / dp$ are determined from equilibrium equations (4). Conditions (6) are equivalent to equation

$$M = 0 \tag{7}$$

where

$$M = (\gamma_1 - p + \gamma_2\zeta_k^2)[\gamma_3 - p - 2\gamma_4\zeta - {}^1/_4(\zeta - \zeta_0)\theta^{-2} + 8\gamma_2\zeta^2 + $$
$$+ {}^1/_{128}\alpha^2\gamma_5(3\zeta_k^2 - \zeta_{0k}^2)] - {}^1/_8[{}^1/_4\zeta_k\theta^{-2} + \gamma_4(2\zeta_k - \zeta_{0k}) - 8\gamma_2(2\zeta_k\zeta - \zeta_0\zeta_{0k})]^2$$

The same Eq. (7) can be obtained if instead of conditions (6) the linear equations of neutral equilibrium are utilized

$$\frac{1}{B}\Delta\Delta\varphi^* = \frac{1}{R}w_{xx}^* + 2w_{xy}w_{xy}^* - w_{xx}w_{yy}^* - w_{xx}^*w_{yy}$$

$$D\Delta\Delta w^* = -\frac{1}{R}\varphi_{xx}^* + \varphi_{yy}w_{xx}^* + \varphi_{xx}w_{yy}^* - 2\varphi_{xy}w_{xy}^* + \varphi_{yy}^*w_{xx} + \varphi_{xx}^*w_{yy} - 2\varphi_{xy}^*w_{xy} \tag{8}$$

where w and φ pertain to the precritical state, i.e. they will be solutions of Eqs. (2), while w^* and φ^* refer to small deviations from this state, and the solution of Eqs. (8) will be sought in the form

$$w^* = {}^1/_2 f^* \cos 2\alpha_1 x + {}^1/_2 f_k{}^* \sin \alpha_1 x \sin \beta_1 y \qquad (9)$$

This result is a consequence of the fact that the stability equations (8) will be variations of nonlinear equations (2). The general theory of this problem is presented in the work of Bolotin [4]. We only note that finding of bifurcation points by means of linear equations of stability (8) mathematically corresponds to finding limit points on curves of equilibrium states obtained on the basis of nonlinear equations (2).

In the calculation of the first limit point in case of large values ζ_0 and ζ_{0k} the condition $M = 0$ does not hold. In this case the unstable branch AB disappears (Fig. 3), while the quantity M for some p has a minimum value which corresponds to the highest rate of change of buckling deformations with respect to load $(d\zeta/dp)_{\max}$ and $(d\zeta_k/dp_{\max})$. The corresponding value of p is taken as the critical value in these cases.

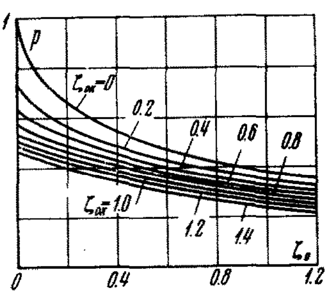


Fig. 4

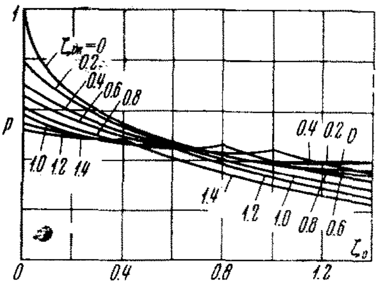


Fig. 5

In Fig. 4 results of calculations of critical stresses are shown as a function of ζ_0 and ζ_{0k}. It is evident that the axially nonsymmetric component of the initial deflection ζ_{0k} has an effect of the same order of magnitude on the value of the limiting load as the symmetrical one. We note that until recently there was no correct notion of this problem. Thus in [5] it was shown that the axially nonsymmetric component has an insignificant influence on the critical loading in comparison to axially symmetric buckling. This result appears as a consequence of simplification of the problem in the analysis.

For a correct quantitative evaluation of stability of a closed shell under longitudinal compression in this manner it is necessary to examine a broader class of perturbations than symmetrical initial deflection.

Let us examine some possiblities for simplification of the problem of determing the critical load. Taking into account that the boundary condition (7) can be obtained by means of equations of neutral equilibrium (8), we shall describe the practical condition of the shell with initial deflection not through nonlinear equations (2) but through linearized equations which we shall obtain from (2) assuming that deflections w differ little from initial deflections w_0. Assuming in (2)

$$w = w_0 + (w - w_0), \quad \varphi = - {}^1/_2\, \sigma h y^2 + \Phi$$

and omitting nonlinear terms which contain $w - w_0$, we obtain the linearized equations

$$\frac{1}{B}\Delta\Delta\Phi = \frac{1}{R}(w - w_0)_{xx} + 2w_{0xy}(w - w_0)_{xy} - w_{0xx}(w - w_0)_{yy} - w_{0yy}(w - w_0)_{xx} \qquad (10)$$

$$D\Delta\Delta(w - w_0) = -\frac{1}{R}\Phi_{xx} - \sigma h w_{xx} + \Phi_{yy}w_{0xx} + \Phi_{xx}w_{0yy} - 2\Phi_{xy}w_{0xy}$$

For determination of the critical value of p, the stability equations in the form (8) are joined to the obtained linearized solution.

Making use of representations (1), (3) and (9) for w_0, w and w^*, we obtain the condition for neutral solution of the equations of stability in the form

$$L = 0 \tag{11}$$

where

$$L = (\gamma_1 - p + \gamma_2\zeta_k^2)\,[\gamma_3 - p - 2\gamma_4\zeta - {}^1/_4\,(\zeta - \zeta_0)\,\theta^{-2} + 8\gamma_2\zeta^2 + {}^1/_{64}\,\gamma_5\,(\zeta_k^2 + \zeta_k\zeta_{0k} - \zeta_{0k}^2)] -$$
$$- {}^1/_8\,[{}^1/_4\,\zeta_k\theta^{-2} + \gamma_4\,(2\zeta_k - \zeta_{0k}) - 8\gamma_2\,(\zeta\zeta_k + \zeta_0\zeta_k - 2\zeta_0\zeta_{0k} + \zeta\zeta_{0k})]^2$$

The critical value of p is determined from condition (11). If however the condition $L = 0$ is not satisfied, then, as before, p is determined from the condition of minimum of the value L.

Results of calculations of critical value p according to condition (11) are presented in Fig. 5. Each curve in Fig. 5 consists of two parts: the first (before the corner point) is determined by the condition $L = 0$, the second by the condition $L_{\min}$. Comparing results of calculations according to conditions $M = 0$ (Fig. 4) and $L = 0$ (Fig. 5), it is possible to draw the conclusion that the linearization of equations of the precritical state gives acceptable results only in a relatively small region of small values of deflections ζ_{0k}. Naturally, for $\zeta_{0k} = 0$ the results of calculations coincide completely because in this case the starting equations are the same.

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LINEAR PROBLEM OF ROTATIONAL OSCILLATIONS OF AN ELASTICALLY COUPLED RIGID SPHERE IN A VISCOUS FLUID, BOUNDED BY A CONCENTRIC STATIONARY SPHERE

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The rotation of a rigid sphere around its diameter with small angular deflection from stationary position is examined under the influence of an elastic force couple in a viscous medium bounded from the outside by a concentric stationary sphere.

The spectrum of oscillations is investigated in detail. The spectral distributions of angular velocity of the sphere are obtained for any positive value of parameters of the